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UNIFIED COUNCIL OLYMPIADS

UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 10
Question Paper Code : UM9267

KEY

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C	D	B	C	A	D	B	D	D	C
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B	D	A	C	C	B	C	B	C	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	C	D	B	D	A	A	C	B	C
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B,C	A,C	B,C	A,B,C,D	A,B,C,D	A	C	A	D	B
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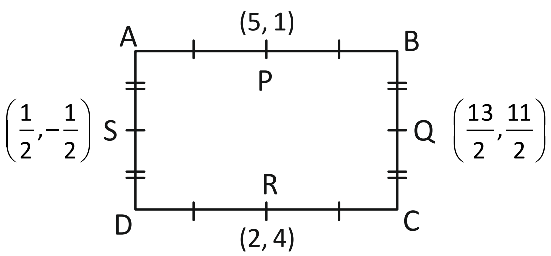
B	B	C	D	C	A	B	D	C	B

EXPLANATIONS

MATHEMATICS - 1

01. (C) $PR = \sqrt{(2-5)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9} = \sqrt{9 \times 2} = 3\sqrt{2}$



$QS = \sqrt{\left(\frac{13}{2} - \frac{1}{2}\right)^2 + \left(\frac{11}{2} + \frac{1}{2}\right)^2} = \sqrt{6^2 + 6^2}$

$= \sqrt{36+36} = \sqrt{36 \times 2} = 6\sqrt{2}$

$\therefore$ Area of the rectangle ABCD

= AB × BC = SQ × PR

$= 6\sqrt{2} \times 3\sqrt{2}$ sq.units

= 36 square units

02. (D) Given $\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$

$$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

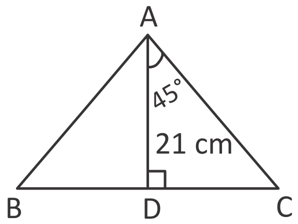
$$\therefore\ \alpha - \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} - \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{-b+\sqrt{b^2-4ac}+b+\sqrt{b^2-4ac}}{2a}$$

$$= \frac{2\sqrt{b^2-4ac}}{2a} = \frac{\sqrt{b^2-4ac}}{a}$$

03. (B) ADC is an isosceles right angled triangle

$\therefore$ AD = DC = r = 21 cm

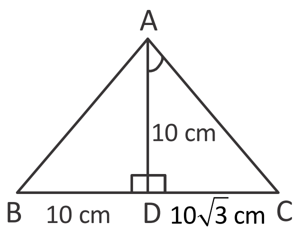


$\therefore$ Volume of the cone $= \dfrac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times 21\times 21\times 21\ \text{cm}^3$$

= 9702 cm^3

04. (C) In ΔABD, $\angle D = 90^\circ \Rightarrow \tan\angle BAD$



$$= \frac{BD}{AD} = \frac{10\text{ cm}}{10\text{ cm}} = 1 = \tan 45^\circ$$

$\therefore\ \angle BAD = 45^\circ$

In ΔADC, $\angle D = 90^\circ \Rightarrow \tan\angle DAC$

$$= \frac{DC}{AD} = \frac{10\sqrt{3}\text{ cm}}{10\text{ cm}} = \sqrt{3} = \tan 60^\circ$$

$\therefore\ \angle DAC = 60^\circ$

$\therefore\ \angle BAC = \angle BAD + \angle DAC = 45^\circ + 60^\circ = 105^\circ$

05. (A) Given tan(A + B) = 1 = tan45°

$\therefore$ A + B = 45° $\rightarrow$ (1)

Given cot (A – B) = $\sqrt{3}$ = cot 30°

$\therefore$ A – B = 30° $\rightarrow$ (2)

$$\begin{array}{rl} A + B = 45^\circ & \rightarrow (1) \\ A - B = 30^\circ & \rightarrow (2) \\ (-)\ \ (+)\ \ \ (-) & \\ \hline 2B = 15^\circ & \end{array}$$

$$B = \frac{15^\circ}{2}$$

06. (D) Let the third vertex be C(x_3, y_3)

Given

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{8}{3}, -6\right)$$

$$\left(\frac{1+6+x_3}{3}, \frac{-2-2+y_3}{3}\right) = \left(\frac{8}{3}, -6\right)$$

$$\frac{7+x_3}{3} = \frac{8}{3} \text{ and } \frac{-4+y_3}{3} = -6$$

$\therefore\ x_3 = 8 - 7 = 1 \quad \therefore\ 4 + y_3 = -18$

$y_3 = -18 + 4 = -14$

$\therefore$ Third vertex(C) = (1, –14)

Distance between A(1, –2) and B(6, –2)

$$AB = \sqrt{(6-1)^2 + (-2+2)^2}$$

$\therefore$ AB = 5

Distance between B(6, –2) and C(1, –14)

$$= \sqrt{(1-6)^2 + (-14+2)^2}$$

$$= \sqrt{5^2 + (-12)^2} = 13$$

BC = 13 cm

Distance between C(1, –14) and A(1, –2)

$$= \sqrt{(1-1)^2 + (-2+14)^2} = 12$$

$\therefore$ BC^2 = AB^2 + AC^2

$\therefore \angle A = 90^\circ$

$\therefore$ Circumcentre is mid point of hypotenuse

$\therefore$ Mid point of

$$BC = \left(\frac{6+1}{2}, \frac{-14-2}{2}\right) = \left(\frac{7}{2}, -8\right)$$

07. (B) Given $20(2x^3 + 3x^2 - 2x)$

$= 20x(2x^2 + 3x - 2)$

$= 20x(2x^2 + 4x - x - 2)$

$= 20x[2x(x + 2) - (x + 2)]$

$= 2 \times 2 \times 5x(x + 2)(2x - 1)$

$45(x^4 + 8x) = 45x(x^3 + 8)$

$= 5 \times 3 \times 3x(x + 2)(x^2 - 2x + 4)$

$\therefore$ LCM $= 4 \times 9 \times 5x(x+2)(2x-1)(x^2-2x+4)$

$= 180x(x + 2)(2x - 1)(x^2 - 2x + 4)$

08. (D) Given A(−1, −1) B(2, 3) and (8, a) are collinear

$\therefore$ Area of $\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|=0$$

$$\therefore \frac{1}{2}\left|-1(3-a)+2(a+1)+8(-1-3)\right|=0$$

$$\therefore \left|-3+a+2a+2-32\right|=0\times 2$$

$$\left|3a-33\right|=0$$

$\therefore$ 3a − 33 = 0

3a = 33

$$a=\frac{33}{3}=11$$

09. (D) Given $2-\sqrt{5}$ is a factor of

$f(x) = (x^4 - 7x^3 + 13x^2 - 5x - 2)$

$\therefore$ $2+\sqrt{5}$ is also a zero of $f(x)$

$\therefore$ $\left(x-2+\sqrt{5}\right)\&\left(x-2-\sqrt{5}\right)$ are the factors of $f(x)$

$$\therefore \left(x-2+\sqrt{5}\right)\left(x-2-\sqrt{5}\right)$$

$$= (x-2)^2-\left(\sqrt{5}\right)^2$$

$= x^2 - 4x + 4 - 5 = (x^2 - 4x - 1)$

is also a factor of $f(x)$

$$
\begin{array}{r|l|l}
x^2-4x-1 & x^4-7x^3+13x^2-5x-2 & x^2-3x+2 \\
 & x^4-4x^3-x^2 & \\
 & (-)\quad(+)\quad(+) & \\
\hline
 & -3x^3+14x^2-5x-2 & \\
 & -3x^3+12x^2+3x & \\
 & (+)\quad(-)\quad(-) & \\
\hline
 & 2x^2-8x-2 & \\
 & 2x^2-8x-2 & \\
 & (-)\quad(+)\quad(+) & \\
\hline
 & 0 &
\end{array}
$$

$x^2 - 3x + 2 = x^2 - 2x - x + 2$

$= x(x - 2) - 1(x - 2)$

$= (x - 2)(x - 1)$

10. (C) Given α, β & γ are the zeros of

$(4x^3 + 5x^2 - 6x)$

$\therefore$ a = 4, b = 5, c = −6 & d = 0

$$\therefore \alpha\beta\gamma = \frac{-d}{a}=\frac{-0}{4}=0$$

11. (B) Let $\frac{1}{x}=a\ \&\ \frac{1}{y}=b$ then $\frac{1}{x}+\frac{1}{2y}=8$

$$\Rightarrow a+\frac{b}{2}=8$$

$$\Rightarrow \frac{2a+b}{2}=8$$

2a + b = 16 $\rightarrow$ (1)

$$\frac{a}{2}-b=-1$$

$$\frac{a-2b}{2}=-1$$

$\therefore$ a − 2b = − 2 $\rightarrow$ (2)

equ (1) × 2 $\Rightarrow$

$$
\begin{array}{l}
4a+2b=32 \\
a-2b=-2 \quad \rightarrow (2) \\
\hline
5a \qquad = 30
\end{array}
$$

$$a=\frac{30}{5}=6$$

2(6) + b = 16 $\rightarrow$ (1)

12 + b = 16

b = 4

$$\therefore a=6=\frac{1}{x}\Rightarrow x=\frac{1}{6}$$

$\therefore\ b = 4 = \frac{1}{y} \Rightarrow y = \frac{1}{4}$

$\therefore\ x - y = \frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = \frac{-1}{12}$

12. (D) Let the usual speed (S_1) be x km/h

$t_1 = \frac{d}{S_1} = \frac{1800\,\text{km}}{x\,\text{km/hr}} = \frac{1800}{x}$ hours

New speed (S_2) = (x + 40) km/hr

$t_2 = \frac{d}{S_2} = \frac{1800}{(x+40)}$h

Given $t_1 - t_2$ = 30 min = $\frac{1}{2}$h

$\left(\frac{1800}{x} - \frac{1800}{x+40}\right)h = \frac{1}{2}h$

$\therefore\ \frac{1800(x+40) - 1800x}{x(x+40)} = \frac{1}{2}$

$\frac{1800(x+40-x)}{x^2+40x} = \frac{1}{2}$

$\therefore\ x^2 + 40x = 1800 \times 40 \times 2$

$x^2 + 40x - 144000 = 0$

$\therefore\ x^2 + 400x - 360x - 144000 = 0$

$x(x + 400) - 360(x + 400) = 0$

$\therefore\ (x + 400)(x - 360) = 0$

$\therefore\ x = -400$ (or) $x = 360$

$\therefore\ x$ = 360 km\hr

[$\therefore\ x = -400$ km\hr is rejected because speed is never negative]

(or)

use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

13. (A) LCM of 32, 36, 40, 45 & 48 = 1440

```
1440) 10000 (6
       8640
      -----
        136
```

$\therefore$ The required least 5 digit number

= 1440 × 7 = 10,080

14. (C) Given $p(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$

$\therefore P(\sqrt{3}) = (\sqrt{3})^4 + 4(\sqrt{3})^3 - 2(\sqrt{3})^2 - 20(\sqrt{3}) - 15$

$= 9 + 12\sqrt{3} - 6 - 20\sqrt{3} - 15$

$P(\sqrt{3}) = -12 - 8\sqrt{3}$

$\therefore\ P(\sqrt{3}) \neq 0$

$P(\sqrt{15}) = (\sqrt{15})^4 + 4(\sqrt{15})^3 - 2(\sqrt{15})^2 - 20(\sqrt{15}) - 15$

$= 225 + 60\sqrt{15} - 30 - 20\sqrt{15} - 15$

$P(\sqrt{15}) = 180 + 40\sqrt{15}$

$\therefore\ P(\sqrt{15}) \neq 0$

$P(-\sqrt{5}) = (-\sqrt{5})^4 + 4(-\sqrt{5})^3 - 2(-\sqrt{5})^2 - 20(-\sqrt{5}) - 15$

$= 25 - 20\sqrt{5} - 10 + 20\sqrt{5} - 15$

= 25 – 25 = 0

$\therefore\ (-\sqrt{5})$ is the zero of the polynomial $P(x)$

15. (C) Given $71x + 37y = 253 \quad \rightarrow (1)$

$37x + 71y = 287 \quad \rightarrow (2)$

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equ (1) + (2) $\Rightarrow$

$108x + 108y = 540$

$108(x + y) = 540$

$x + y = \frac{540}{108} = 5$

$x + y = 5 \quad \rightarrow (3)$

$37x + 71y = 287 \rightarrow (2)$

equ (3) × 37 $\Rightarrow$ $37x + 37y = 185$

(–) (–) (–)

$34y = 102$

$y = \frac{102}{34} = 3$

$x + 3 = 5 \rightarrow (3)$

$x = 2$

$\therefore\ x - y = 2 - 3 = -1$

16. (B) Given $P(x) = (b - c)x^2 + (c - a)x + (a - b)$

$P(1) = (b - c)(1)^2 + (c - a)(1) + (a - b)$

$= b - c + c - a + a - b$

$P(1) = 0 \Rightarrow (x - 1)$ is a factor of $P(x)$

Given the roots are equal

$\therefore$ other root $= (x - 1)$

$\therefore$ Product of the roots $= \dfrac{(a-b)}{(b-c)} = 1 \times 1$

$a - b = b - c$

$a + c = b + b$

$\therefore 2b = c + a$

17. (C) Given 2023, 2016, 2009, 2002, ...1869 are in AP

$\therefore a = 2023$ & $d = a_2 - a_1 = 2016 - 2023 = -7$

But $a_n = a + (n - 1)d = 1869$

$2023 + (n - 1)(-7) = 1869$

$(n - 1)(-7) = -154$

$(n - 1) = \dfrac{154}{7} = 22$

$n = 22 + 1 = 23$

$\therefore$ Middle term

$= \dfrac{1}{2}(23+1) = \dfrac{24}{2} = 12^{th}$ term

$\therefore a_{12} = a + 11d = 2023 + 11(-7)$

$= 2023 - 77 = 1946$

18. (B) If $\theta = 45°$ then LHS $= 1 + \cos^2 45°$

$= 1 + \left(\dfrac{1}{\sqrt{2}}\right)^2 = 1 + \dfrac{1}{2} = \dfrac{3}{2}$

RHS $= 3\cos 45° \sin 45° = 3 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = \dfrac{3}{2}$

19. (C) Given $3\pi r^2 = 4158\ cm^2$

$3 \times \dfrac{22}{7} \times r^2 = 4158\ cm^2$

$r^2 = 4158 \times \dfrac{1}{3} \times \dfrac{7}{22} cm^2$

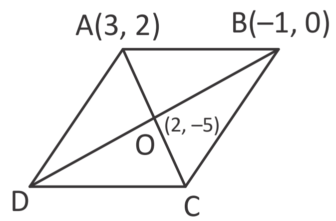
$r = \sqrt{3 \times 3 \times 7 \times 7\ cm^2}$ $\qquad$ $r = 21$ cm

$\therefore$ Volume $= \dfrac{2}{3}\pi r^3 = \dfrac{2}{3} \times \dfrac{22}{7} \times 21 \times 21 \times 21\ cm^3$

$= 19,404\ cm^3$

20. (B) Given the vertices of

ΔAOB are $A(3, 2)$ $O(2, -5)$ & $B(-1, 0)$



$\therefore$ Area of ΔAOB

$= \dfrac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$= \dfrac{1}{2}\left|3(-5-0) + 2(0-2) - 1(2+5)\right|$

$= \dfrac{1}{2}\left|-15-4-7\right|$

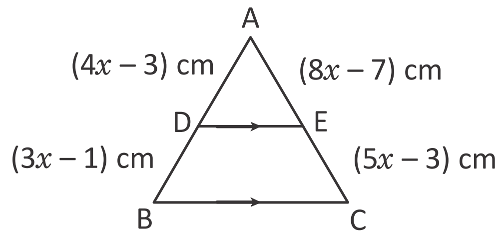
$= \dfrac{1}{2}\left|-26\right|$ units^2

$= \dfrac{1}{2} \times 26$ units

$\therefore$ Area of the parallelogram ABCD

$= 4 \times$ area of $\Delta AOB = 52$ units^2

21. (A) In ΔABC, given DE || BC



$\therefore \dfrac{AD}{DB} = \dfrac{AE}{EC}$ $[\because$ Thales theorem$]$

$\Rightarrow \dfrac{(4x-3)cm}{(3x-1)cm} = \dfrac{(8x-7)cm}{(5x-3)cm}$

$\Rightarrow (4x - 3)(5x - 3) = (8x - 7)(3x - 1)$

$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$\therefore$ $24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$

$4x^2 - 2x - 2 = 0$

$2(2x^2 - x - 1) = 0$

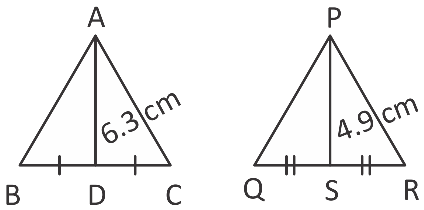
$$2x^2 - 2x + x - 1 = \frac{0}{2}$$

$$2x(x - 1) + 1(x - 1) = 0$$

$$(x - 1)(2x + 1) = 0$$

$\therefore$ $x = 1$ (or) $x = \dfrac{-1}{2}$ is rejected because length is never negative

22. (C) Given $\Delta ABC \sim \Delta PQR$



$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

$\Delta ABC \sim \Delta PQR$

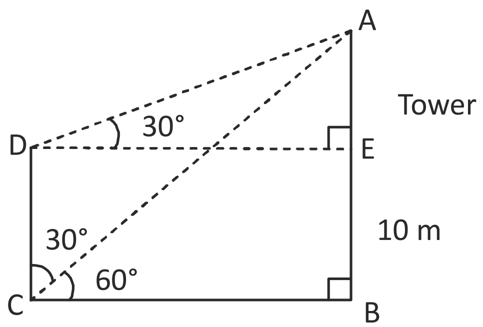
$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \frac{81\text{cm}^2}{\text{area of } \Delta PQR} = \left(\frac{6.3\text{ cm}}{4.9\text{ cm}}\right)^2 = \frac{0.9}{0.7}$$

$$\Rightarrow \frac{81\text{cm}^2}{\text{area of } \Delta PQR} = \left(\frac{9}{7}\right)^2 = \frac{81}{49}$$

$$\therefore \text{ Area of } \Delta PQR = 81\text{ cm}^2 \times \frac{49}{81} = 49\text{ cm}^2$$

23. (D) In ΔADE, $\angle E = 90° \Rightarrow \tan 30° = \dfrac{AE}{DE}$



$$\frac{1}{\sqrt{3}} = \frac{AE}{DE}$$

$$DE = \sqrt{3}\,AE \qquad \rightarrow (1)$$

In ΔABC, $\angle B = 90° \Rightarrow \tan 60° = \dfrac{AB}{BC}$

$$\sqrt{3} = \frac{AE + 10\text{ m}}{DE} \quad [\because BE = DC \;\&\; DE = BC]$$

$$\therefore \sqrt{3}\,DE = AE + 10\text{ m}$$

$$\therefore \sqrt{3} \times \sqrt{3}\,AE = AE + 10\text{m} \; [\because \text{from equ}(1)]$$

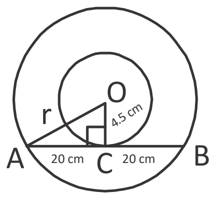
$$3AE - AE = 10\text{ m} \; [\because \text{from equ}(1)]$$

$2AE = 10$ m

$AE = 5$ m

$\therefore$ $AB = AE + EB = 15$ m

24. (B) Given $OC = 4.5$ cm , $OA = r$ cm $OC \perp AB$



$$AC = BC = \frac{AB}{2} = \frac{40\text{ cm}}{2} = 20\text{ cm}$$

In ΔAOC, $\angle C = 90° \Rightarrow AO^2 = AC^2 + OC^2$

$= (20)^2 + (4.5)^2$

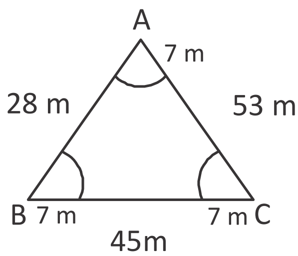
$= 400 + 20.25$

$AO = \sqrt{420.25\text{ cm}^2}$

$AO = 20.5$ cm

25. (D) Area of the field grazed by three horses

$$= \frac{\angle A}{360°} \times \pi r^2 + \frac{\angle B}{360°} \times \pi r^2 + \frac{\angle C}{360°} \times \pi r^2$$



$$= \frac{\pi r^2}{360°}(\angle A + \angle B + \angle C)$$

$$= \frac{22}{7} \times 7 \times 7\,cm^2 \times \frac{1}{360°} \times 180°$$

[$\because \angle A + \angle B + \angle C = 180°$]

$= 77\ m^2$

$$s = \frac{a+b+c}{2} = \frac{28\,m + 45\,m + 53\,m}{2}$$

$$= \frac{126\,m}{2} = 63\,m$$

Area of the triangular field

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{63 \times (63-28)(63-45)(63-53)}$$

$$= \sqrt{9 \times 7 \times 35 \times 18 \times 10}$$

$$= \sqrt{9 \times 7 \times 7 \times 5 \times 2 \times 9 \times 2 \times 5}$$

$= 9 \times 7 \times 5 \times 2\ m^2$

$= 630\ m^2$

$\therefore$ Area of the field that the horses cann't be grazed $= 630\ m^2 - 77\ m^2 = 553\ m^2$

26. (A) Total surface area of the solid

$= 2\pi r^2 + 2\pi rh + 2\pi r^2$

$= 2\pi r\ (r + h + r)$

$= 2\pi r(h + 2r)$

$$= 2 \times \frac{22}{7} \times 15\,cm \times 87.5\,cm$$

$= 8250\ cm^2$ [$\because$ Given $h + 2r = 87.5$ cm]

$\therefore$ Total cost of polishing

$$= 8250\,cm^2 \times \frac{20\,paise}{1\,cm^2}$$

= 165000 paise

= ₹ 1650

27. (A) Given $S_5 + S_7 = 167$

$$\frac{5}{2}(2a+4d) + \frac{7}{2}(2a+6d) = 167$$

$$\frac{5}{2} \times 2(a+2d) + \frac{7}{2} \times 2(a+3d) = 167$$

$5a + 10d + 7a + 21d = 167$

$12a + 31d = 167 \qquad \rightarrow(1)$

Given $S_{10} = 235$

$$\frac{10}{2}[2a+9d] = 235$$

$$2a + 9d = \frac{235}{5} = 47 \qquad \rightarrow(2)$$

equ (2) $\times 6 \Rightarrow 12a + 54d = 282$

$\ 12a + 31d = 167 \quad \rightarrow (1)$

$\ (-) \quad (-)$

$\ 23\ d = 115$

$$d = \frac{115}{23} = 5$$

$2a + 9(5) = 47 \qquad \rightarrow(2)$

$2a + 45 = 47$

$2a = 47 - 45 = 2$

$a = 1$

$\therefore a_{10} = a + 9d = 1 + 9(5) = 46$

28. (C) Distance between (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (a\sin\theta - b\cos\theta - 0)^2}$$

$$= \sqrt{(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2}$$

$$= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta}$$

$$= \sqrt{a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$$

$$= \sqrt{a^2 + b^2}$$

29. (B) BQ = BP = 8 cm & AR = AP = 7 cm

[$\because$ length of the tangents drawn from an external point are equal]

$\therefore$ CR = 15 cm – 7 cm = 8 cm = CQ

$\therefore$ BC = BQ + QC = 8 cm + 8 cm = 16 cm

30. (C) Given LHS

$$=\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\frac{1}{\sin^2\theta}}+\frac{1}{1+\frac{1}{\cos^2\theta}}$$

$$=\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{\left(\frac{\sin^2\theta+1}{\sin^2\theta}\right)}+\frac{1}{\left(\frac{\cos^2\theta+1}{\cos^2\theta}\right)}$$

$$=\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{\sin^2\theta}{1+\sin^2\theta}+\frac{\cos^2\theta}{1+\cos^2\theta}$$

$$=\frac{1}{1+\sin^2\theta}+\frac{\sin^2\theta}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{\cos^2\theta}{1+\cos^2\theta}$$

$$=\left(\frac{1+\sin^2\theta}{1+\sin^2\theta}\right)+\left(\frac{1+\cos^2\theta}{1+\cos^2\theta}\right) = 2$$

MATHEMATICS - 2

31. (B, C)

Let $\sqrt{\frac{x}{1-x}}$ be $y \Rightarrow y+\frac{1}{y}=\frac{13}{6}$

$$\Rightarrow \frac{y^2+1}{y}=\frac{13}{6}$$

$\Rightarrow 6y^2 - 13y + 6 = 0$

$6y^2 - 9y - 4y + 6 = 0$

$3y(2y - 3) - 2(2y - 3) = 0$

$$y=\frac{2}{3} \text{ (or) } \frac{3}{2}$$

$$\sqrt{\frac{x}{1-x}}=\frac{2}{3}\text{(or)}\sqrt{\frac{x}{1-x}}=\frac{3}{2}$$

squaring on both sides

$\frac{x}{1-x}=\frac{4}{9}$	$\frac{x}{1-x}=\frac{9}{4}$
$9x = 4 - 4x$	$4x = 9 - 9x$
$13x = 4$	$13x = 9$
$x=\frac{4}{13}$	$x=\frac{9}{13}$

32. (A, C)

Given:- In ΔABC, AD is a median

Construction:- AE $\perp$ BC

Proof:- In ΔABE, $\angle$E = 90°

$\Rightarrow AB^2 = AE^2 + BE^2$

$= AE^2 + (BD - ED)^2$

$= AE^2 + BD^2 + ED^2 - 2BD \times ED$

$= AE^2 + ED^2 + BD^2 - 2BD \times ED$

$\therefore AB^2 = AD^2 + BD^2 - 2BD \times DE \quad \rightarrow (1)$

Similarly $AC^2 = AD^2 + DC^2 + 2DC \times DE$

$= AD^2 + BD^2 + 2\, BD \times DE \quad \rightarrow (2)$

[$\because$ DC = BD]

equ (1) + (2) $\Rightarrow AB^2 + AC^2 = 2AD^2 + 2BD^2$

$= 2(AD^2 + BD^2)$

$= 2(AD^2 + DC^2)$

33. (B, C)

Given $\Delta < 0$

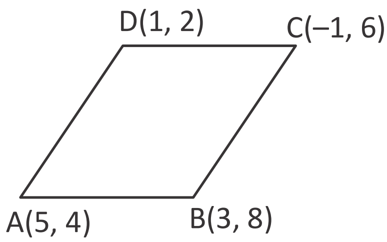
$(-6)^2 - 4(k)(-1) < 0$

$36 + 4k < 0$

If $k = -10$ then $36 + 4k < 0$

If $k = -1000$ then $36 + 4k < 0$

34. (A, B, C, D)



$$\text{Mid point of AC} = \left(\frac{5-1}{2},\frac{4+6}{2}\right)=(2,5)$$

$$\text{Mid point of BD} = \left(\frac{1+3}{2},\frac{2+8}{2}\right)=(2,5)$$

$\therefore$ Diagonals bisect each other

$\therefore$ ABCD is a parallelogram

$\therefore$ Every parallelogram is a trapezium

$\Rightarrow$ ABCD is a trapezium

35. (A, B, C, D)

Given $3x + 4y = 5$ $\rightarrow$ (1)

$0.06x + 0.08y = 0.1$ $\rightarrow$ (2)

equ (2) × 100 $\Rightarrow$ $6x + 8y = 10$ $\rightarrow$ (2)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

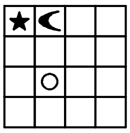
$\Rightarrow$ Both lines are coinciding lines

$$\therefore \left(0, \frac{5}{4}\right), \left(\frac{5}{3}, 0\right), (1, 0.5) \,\&\, (15, -10)$$

are the solutions of the given equations

REASONING

36. (A) The frames in this question are divided into 16 squares, around which three shapes move, each in its own pattern.



The ★ moves between the four corners of the frames, clockwise. The ☾ moved up and left alternately as if going up the stairs. The ○ moved upwards in diagonal movements, up and left, up and right, etc., and when it reaches the top of the frame, it moves down again in the same manner.

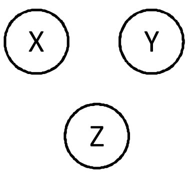
37. (C) $\boxed{3a}$, (2b), 7c $\rightarrow$ (Truth) $\boxed{is}$ eternal

7c, **9a**, 8b, $\boxed{3a}$ $\rightarrow$ Enmity $\boxed{is}$ **not** eternal

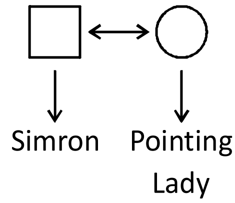
9a, 4d, (2b), 8a $\rightarrow$ (Truth) does **not** perish

8b is coded as enmity

38. (A)



39. (D) Cousin sister



40. (B) Option (A) : 8 × 7 – 6 + 9 = 59 ≠ 25 (✗)

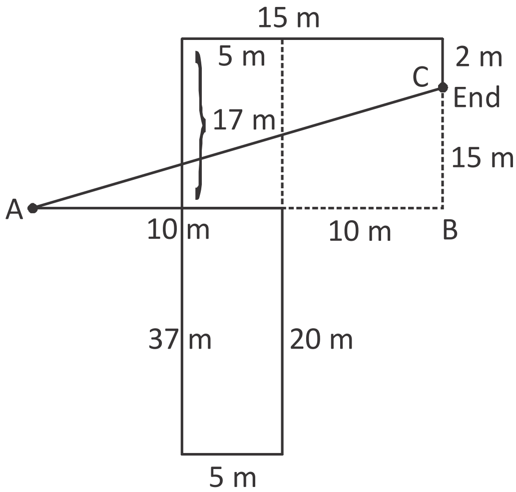
Option (B) : 7 + 8 – 3 × 5 = 0 = 0 (✓)

Option (C) : 6 – 7 × 2 + 8 = 16 ≠ 35 (✗)

Option (D) : 7 × 2 – 8 + 6 = 12 ≠ 9 (✗)

41. (B) ARCHLO = CHORAL (composed for or sung by a choir or chorus.

42. (B)



AB = 10 + 10 = 20 M

AC = ?

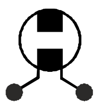
BC = 15 M

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

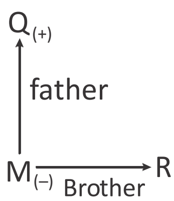
$$= \sqrt{(20)^2 + (15)^2}$$

$$= \sqrt{400 + 225} = \sqrt{625} = 25\text{m North East}$$

43. (C)



44. (D)



45. (C) Given that six products - Airel, vivel, Rin, Nirma, Gillette Gel and Pepsodent

From 1st condition: Rin and Ariel are next to each other.

From 2nd condition at least two products are between Ariel & Nirma

From 3rd condition pepsodent is kept between Gillette Gel and Rin, and at least 2 products between Gillette Gel and Rin, and at least 2 products between Pepsodent and Vivel.

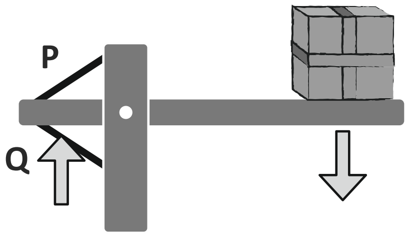
From 5th condition Vivel is not kept at 1st window

Final arrangement would be Nirma Gillette Pepsodent Rin Ariel Vivel

CRITICAL THINKING

46. (A) Educating the school going children on politics will definitely acquaint them with the intricacies and modalities of the same and will help them in making an informed decision while casting their vote. Thus, I hold strong. II is vague and does not give any argument.

47. (B) The box is placed on the shelf and weighs downward and hence rod Q is supporting the box.



48. (D) When you traced your path on the map, you should have seen that if officer Rawath is heading east on main street and he makes a U-turn, he will be heading west. If he turns onto third street, the only way he can turn will be north on third street. If he makes a s econd U-turn, he will now be facing south.

49. (C) Both statements I and II are effects of a common cause.

50. (B) Q and S

The End